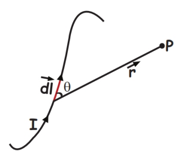
MOVING CHARGES AND MAGNETISM

1

Biot-Savart's Law

$dB \propto \frac{Idl\sin\theta}{r^2}$

$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$



$\mu_0 \rightarrow$ magnetic permeability of free space or vaccum

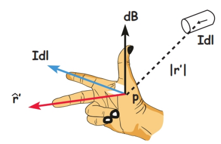
$\mu_0 = 4\pi \times 10^{-7}$

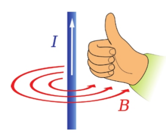
In vector form,

$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times \vec{r}}{r^3}$

$d\vec{B}$ is perpendicular to both $\vec{dl}$ and $\vec{r}$. BY using right hand screw rule we can find direction of magnetic field

Here, B is into the plane





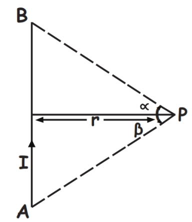
Magnetic field circulates around the current carrying wire

Formula of Field due to straight wire

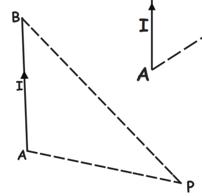
1) At point P

B is into the plane

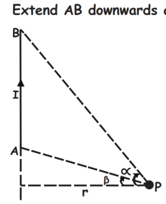
$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$, ⊗



2)



Extend AB downwards and draw a perpendicular from P



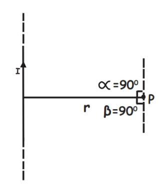
$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha - \sin\beta)$, ⊗

3) Wire of infinite length

$\alpha = 90$ and $\beta = 90$

$B = \frac{\mu_0 I}{4\pi r}(\sin 90 + \sin 90)$

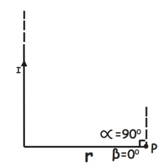
$= \frac{\mu_0 I}{2\pi r}$, ⊗



4) Wire of semi infinite length

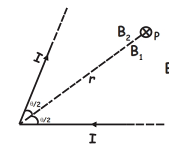
$B = \frac{\mu_0 I}{4\pi r}(\sin 90 + \sin 0)$

$= \frac{\mu_0 I}{4\pi r}$

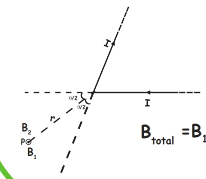


$B(P_1) = B(P_2) = 0$

Field Due to bent wire

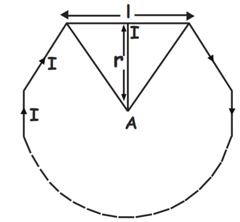


$B_{total} = B_1 + B_2 = 2\times \frac{\mu_0 I}{4\pi r}\frac{[1+\cos\theta/2]}{\sin\theta/2}$, ⊙



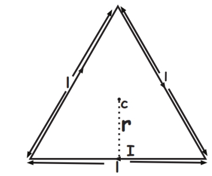
$B_{total} = B_1 + B_2 = 2\times \frac{\mu_0 I}{4\pi r}\frac{[1-\cos\theta/2]}{\sin\theta/2}$ ⊗

Field due to polygon



$B_A = \frac{\mu_0 n I}{\pi l}\tan\frac{\pi}{n}\sin\frac{\pi}{n}$

eg: equilateral triangle

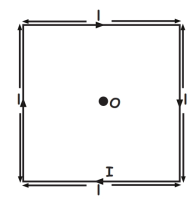


$B_c = \frac{9}{2}\frac{\mu_0 I}{\pi l}$

If a total lenght l is bent as an equilateral triangle, each side has length l/3 then,

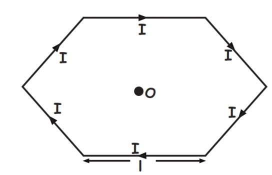
$B' = \frac{27}{2}\frac{\mu_0 I}{\pi l}$

Square



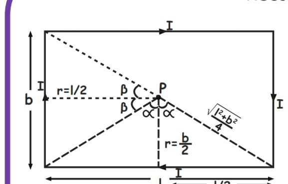
$B_o = \frac{2\sqrt{2}\,\mu_0 I}{\pi l}$

Hexagon



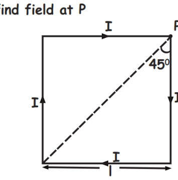
$B_o = \frac{\sqrt{3}\,\mu_0 I}{\pi l}$

Rectangle



$B_p = \frac{2\mu_0 I}{\pi}\left(\frac{\text{diagonal}}{\text{area}}\right) = 2\mu_0 I\frac{\sqrt{l^2+b^2}}{lb}$

Note:- To find field at P



Here P is a point at vertex

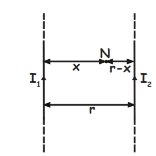
$B_p = \frac{\mu_0 I}{2\sqrt{2}\,\pi l}$

Neutral Points

Points at which magnetic field becomes zero are called neutral points.

Case 1: Parallel wires carrying current in same direction $(I_1 > I_2)$

$\therefore x = \frac{I_1 r}{I_1 + I_2}$

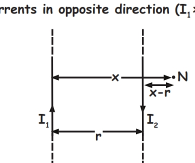


If y is the distance from 2[nd] conductor then

$y = \frac{I_2 r}{I_1 + I_2}$

Case 2: Parallel wires carrying currents in opposite direction $(I_1 > I_2)$

$x = \frac{I_1 r}{I_1 - I_2}$

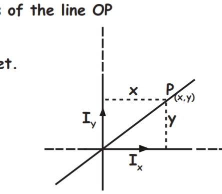


Case 3: Wires perpendicular to each other

Field wll be zero on all points of the line OP

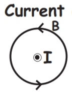
Comparing with y=mx+C we get.

slope $m = \frac{I_x}{I_y}$



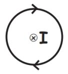
Wire perpendicular to plane

1) Current directed out of plane



B is in anticlockwise direction

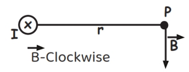
2) Current directed into the plane



B is in clockwise direction

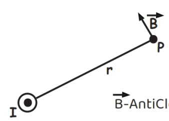
Direction of field

1)



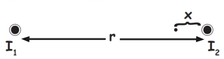
$B = \frac{\mu_0 I}{2\pi r}$

2)



$\vec{B} = \frac{\mu_0 I}{2\pi r}$

Neutral Point

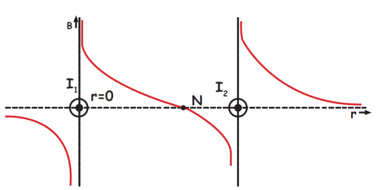


$I_1 > I_2$

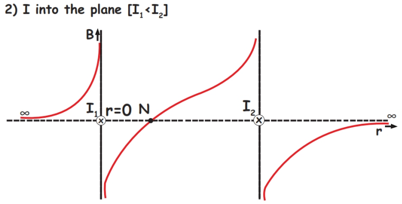
$x = \frac{I_1 r}{I_1 + I_2}$

Graphs

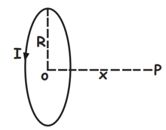
1) I out of the plane, $I_1 > I_2$



2) I into the plane $[I_1 < I_2]$



Field due to a circular ring & Field at the axis of a ring



$$B_P = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

Every current carrying loop acts as a magnetic dipole

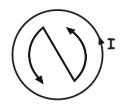
$\vec{M} = I\vec{A}$

Direction of A is determined using right hand thumb rule

Here, $\vec{M} = I\vec{A} \Rightarrow M = I\pi R^2$

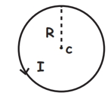
$$B = \frac{\mu_0}{4\pi}\,\frac{2M}{(x^2+R^2)^{3/2}}$$

If current is flowing in anticlockwise direction



If current is flowing in clockwise direction

Field at the centre of ring

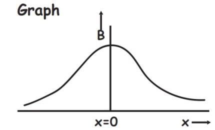


$B_c = \frac{\mu_0 I}{2R}$

Field at the axis in terms of field at center

$$B_{axis} = \frac{B_c}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

Graph



Percentage change in field with respect to centre

for the points on the axis

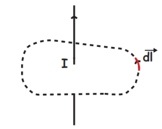
% change in B = $1-(1+x^2/R^2)^{-3/2} \times 100\%$

For change < 10% $= \left(\frac{3}{2} \times \frac{x^2}{R^2}\right) \times 100\%$

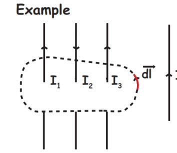
Ampere's Cicutal Law

The line integral of magnetic field over a closed loop is equal to μ_0 times the total current enclosed by the loop

$\oint \vec{B}.\vec{dl} = \mu_0 I_{enclosed}$

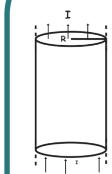


Example

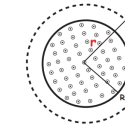


$\oint \vec{B}.\vec{dl} = \mu_0 [I_1 - I_2 + I_3]$

Solid Cylinder



Outside : r > R



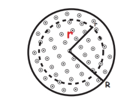
$B_{out} = \frac{\mu_0 I}{2\pi r}$

$B_{out} \propto \frac{1}{r}$

On the surface r = R

$B_s = \frac{\mu_0 I}{2\pi R}$

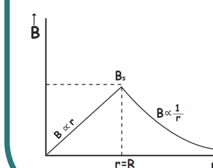
Inside (r < R)



$B_{in} = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$

$B_{in} \propto r$

Graph

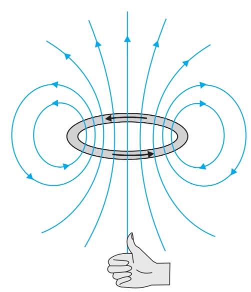


Note

$B_{out} = B_s \frac{R}{r}$

$B_{in} = B_s \frac{r}{R}$

Magnetic field lines for a current loop

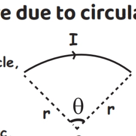


If there are N loops, $B = \frac{\mu_0 N I R^2}{2(x^2+R^2)^{3/2}}$

Field at the Centre due to circular arc

Field at centre of full Circle,

$B = \frac{\mu_0 I}{2r}$

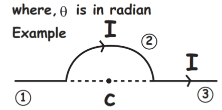


Then field at centre of arc,

$B = \frac{\mu_0 I}{4\pi}\,\frac{\theta}{r}$

where, θ is in radian

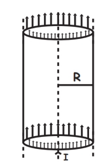
Example



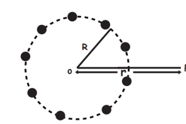
$B_c = B_1 + B_2 + B_2$

$= \frac{\mu_0 I}{4\pi}\,\frac{\pi}{r} = \frac{\mu_0 I}{4r}$

Hollow cylinder (pipe)



Outside : r > R



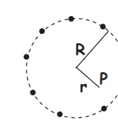
$B_{out} = \frac{\mu_0 I}{2\pi r}$

$B_{out} \propto \frac{1}{r}$

On the surface : r = R

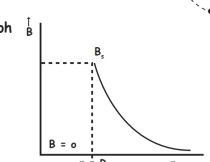
$B_s = \frac{\mu_0 I}{2\pi R}$

Inside : r < R



$B_{in} = 0$

Graph



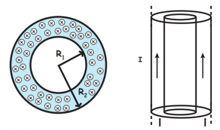
MOVING CHARGES AND MAGNETISM

2

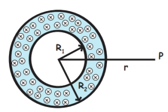
MOVING CHARGES AND MAGNETISM

3

Annular pipe

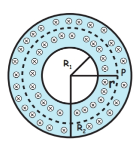


- Outside ($r > R_2$)



$$B_{out} = \frac{\mu_0 I}{2\pi r}$$

- In between $R_1 < r < R_2$



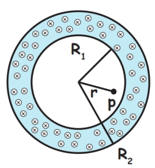
$$I_{enc} = \frac{I}{\pi (R_2^2 - R_1^2)} \times \pi (r^2 - R_1^2)$$

$$B_{btw} = \frac{\mu_0}{2\pi r} \times I \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$$

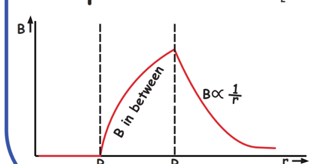
Inside

$r < R_1$

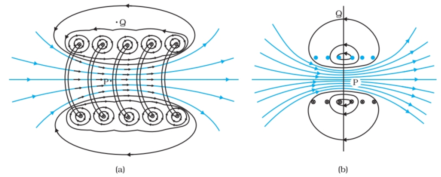
$B_{in} = 0$

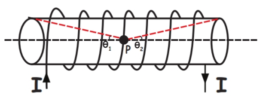


Graph



Solenoid





$$B_p = \frac{1}{2} \mu_0 n I (\cos\theta_1 + \cos\theta_2)$$

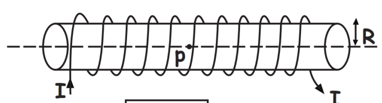
where $n \longrightarrow$ no.of turns per unit length

Tightly packed long solenoid

Long solenoid: Radius is small compared to the length of the solenoid

For a long solenoid the middle region will have a uniform magnetic field

1) At centre



$B_{out} = 0$

$B = \mu_0 n I$, Along the axis

2) End point

$\theta_1 = 0 \quad \theta_2 = 90^\circ$

$$B_{end} = \frac{1}{2} \mu_0 n I = \frac{B_{axis}}{2}$$

Charged particle in magnetic field

Magnetic Force on particle

$\vec{F}_m = q(\vec{v} \times \vec{B})$

Case $\vec{v} \perp \vec{B}$

Path: Circle

Radius of circular path

i) $F_m = \frac{mv^2}{r}$

$\Rightarrow qvB = \frac{mv^2}{r}$

$\Rightarrow r = \frac{mv}{qB}$

ii) Momentum

$p = mv = \sqrt{2mk}$

$\Rightarrow r = \frac{\sqrt{2mk}}{qB}$

If charged particle at rest is accelerated by a voltage of 'V' volt then,

K.E $= qV$

$r = \frac{\sqrt{2mqV}}{qB}$

If $K_i \neq 0$, then $K_f = qV + K_i$

$r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2m(qV + K_i)}}{qB}$

Time Period

$$T = \frac{2\pi m}{qB}$$

$\Rightarrow T \propto r^0$

$T \propto v^0$

We have $\frac{q}{m}$ = specific charge

$\Rightarrow T \propto \frac{1}{\text{specific charge}}$

Calculation of ratio of radii

1) All particles are projected with same speed:

V=Constant

$r = \frac{mv}{qB} \Rightarrow r \propto \frac{m}{q}$

$r_p : r_d : r_\alpha = \frac{1}{e} : \frac{2}{e} : \frac{4}{2e} = 1:2:2$

2) All particles are projected with same momentum

$r = \frac{p}{qB} \Rightarrow r \propto \frac{1}{q}$

$r_p : r_d : r_\alpha = \frac{1}{e} : \frac{1}{e} : \frac{1}{2e} = 2:2:1$

As radius ↑ Curvature ↓

3) All particles are projected with same kinetic energy

$r = \frac{mv}{qB} = \frac{\sqrt{2mk}}{qB}$

$\Rightarrow r \propto \frac{\sqrt{m}}{q}$

$r_p : r_d : r_\alpha = \frac{\sqrt{1}}{e} : \frac{\sqrt{2}}{e} : \frac{\sqrt{4}}{2e}$

$1 : \sqrt{2} : 1$

4) All particles are projected by same accelerating potential

$r = \frac{\sqrt{2mqV}}{qB} \Rightarrow r \propto \sqrt{\frac{m}{q}}$

$r_p : r_d : r_\alpha = \sqrt{\frac{1}{e}} : \sqrt{\frac{2}{e}} : \sqrt{\frac{4}{2e}} = 1 : \sqrt{2} : \sqrt{2}$

Helical path & pitch

$\vec{V}$ makes angle θ with $\vec{B}$ ($\theta \neq 0, \pi, \pi/2$)

Path of charge is Helical

$\vec{V}$ < $\vec{V}_{\parallel} \rightarrow$ || to $\vec{B}$; $\vec{V}_{\perp} \rightarrow \perp$ to $\vec{B}$

1) Radius

$R = \frac{mv_\perp}{qB}$

$R = \frac{mv\sin\theta}{qB} = \frac{\sqrt{2mqV}}{qB} \sin\theta$

2) Time period

$T = \frac{2\pi m}{qB}$

3) Pitch $= V_{\parallel} \times T$

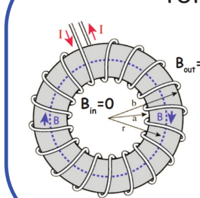
$= V\cos\theta \times \frac{2\pi m}{qB}$

$= 2\pi \left(\frac{mv}{qB}\right) \cos\theta$

$= 2\pi \frac{\sqrt{2mK}}{qB} \cos\theta$

$= 2\pi \frac{\sqrt{2mqV}}{qB} \cos\theta$

Toroid



r is the mean radius

$r = \frac{a_1 + b_2}{2}$

$B_{out} = 0$

$B_{inside} = 0$

$B_{in\ between} = \mu_0 n I$

unit and dimension of B

$F = qvB\sin\theta$

$B = \frac{F}{qv\sin\theta}$

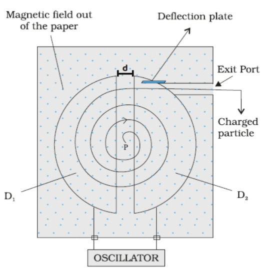
$[B] = \frac{MLT^{-2}}{AT \times LT^{-1}} = MA^{-1}T^{-2}$

Unit $\rightarrow$ $kgA^{-1}s^{-2}$ = Tesla (T)

MOVING CHARGES AND MAGNETISM

4

Cyclotron



1) Maximum kinetic energy

$K_{max} = \frac{q^2B^2R^2_{dee}}{2m}$

2) Number of oscillations ,N

Work done=ΔK

$NxqEd = \frac{q^2B^2R^2}{2m}$

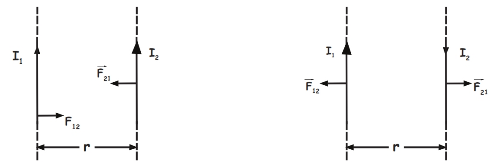
E is in the plane of the dees

3) Time period of revolution= $\frac{2\pi m}{qB}$

4) Cyclotron frequency $\nu_c = \frac{qB}{2\pi m}$

Force between parallel conductors

1) Force between two long parallel current conductors



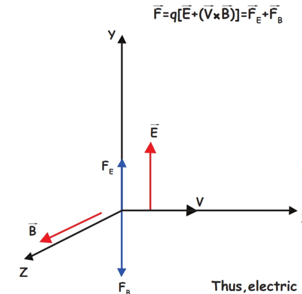
Parallel currents attract each other. Anti parallel currents repel each other.
Force per unit length of conductor

$F_{12} = F_{21} = \frac{\mu_0}{2\pi}\frac{I_1I_2}{r}$

Net force on 'l' length of conductor= $\frac{\mu_0}{4\pi}\frac{2I_1I_2}{r} \times l$

Motion of charged particle in Crossed Electric and Magnetic field ($\vec{E} \perp \vec{B}$)[velocity selector]

$\vec{F} = q[\vec{E} + (\vec{V} \times \vec{B})] = \vec{F}_E + \vec{F}_B$



$\vec{F}_E = q\vec{E} = qE\hat{j}$

$\vec{F}_B = q(\vec{V} \times \vec{B}) = q(V\hat{i} \times B\hat{k})$

$= -qVB\hat{j}$

$\therefore \vec{F} = q(E - VB)\hat{j}$

If V= E/B , F_{net} = 0

Charge moves in a Straight line

Thus, electric and magnetic forces are in opposite directions as shown in figure.

Force on current carrying Conductor in magnetic field

consider a small element $d\vec{l}$ of the conductor

$d\vec{F} = I(d\vec{l} \times \vec{B})$

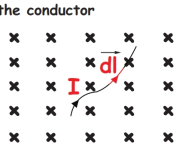
To find resulting force

$\int d\vec{F} = \int I(d\vec{l} \times \vec{B})$

$= I\int d\vec{l} \times \vec{B}$

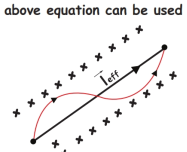
In uniform field

$\vec{F} = I(\int d\vec{l}) \times \vec{B} = I(\vec{l} \times \vec{B})$



The above equation can be used in the following situations

1)



$\vec{F} = I(\vec{l}_{eff} \times \vec{B})$

$= I\, l_{eff} B\sin\theta$

2) $\vec{l}_{eff}$ perpendicular to $\vec{B}$

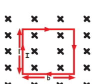
θ=90

$F_{max} = I\, l_{eff} B$

For closed Loop in uniform field

Here, l_{eff}=0

$\therefore \vec{F}$=0



Note:

1) The constant, $\frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.

2) Figure of merit of a galvanometer

$G = \frac{I}{\Phi} = \frac{K}{NAB}$

Lorentz force

A charge q in an electric field $\vec{E}$ experiences the electric force

$\vec{F} = q\vec{E}$

The magnetic force experienced by the charge q moving with velocity $\vec{V}$ in the magnetic field $\vec{B}$ is given by

$\vec{F}_m = q(\vec{V} \times \vec{B})$

The total force,or the Lorentz force,experienced by the charge q due to both electric and magnetic field is given by

$$\vec{F} = q[\vec{E} + (\vec{V} \times \vec{B})]$$

Moving Coil Galvanometer (MCG)

τ= NIABsinθ

Sensitivity of a Galvanometer

Current sensitivity, $I_s = \frac{\Phi}{I} = \frac{NAB}{K}$

Where K→torsional constant

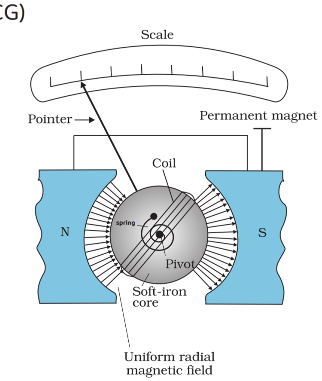
Φ→ deflection of galvanometer

Voltage Sensitivity

Voltage Sensitivity, $V_s = \frac{\Phi}{V} = \frac{\Phi}{IR}$

$= \frac{NAB}{KR}$

Voltage Sensitivity= $\frac{\text{Current Sensitivity}}{R}$



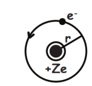
Magnetic dipole moment of a revolving electron

$I = \frac{e}{T}$

T is the time period of revolution

$T = \frac{2\pi r}{V}$

$I = \frac{eV}{2\pi r}$



There will be a magnetic moment,usually denoted by μ associated with this circulating current

$\mu = I\pi r^2 = \frac{eVr}{2}$

$\mu = \frac{e}{2m_e}(m_e vr) = \frac{e}{2m_e} L$

$\vec{L} = \vec{r} \times \vec{P}$

$= rmv\sin\theta$

$= mvr$

In vector form

$\vec{\mu} = \frac{-e}{2m_e}\vec{L}$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment

The ratio of magnetic moment to the angular momentum is called gyromagnetic ratio

$\frac{\mu}{L} = \frac{e}{2m_e}$

Its value is a constant and is equal to 8.8×10^{10} C/kg for an electron

According to Bohr's quantization condition, angular momentum assumes a discrete set of values, namely.

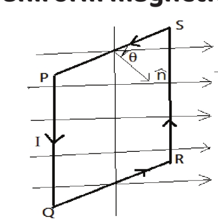
$L = \frac{nh}{2\pi}$ where n= 1, 2, 3.....

h→planck's constant

Taking n=1, we get,

$\mu_{min} = \frac{e}{4\pi m_e}h = 9.27 \times 10^{-24}$ Am²

Torque on a current loop in a uniform magnetic field



Then τ = NIABsinθ

Special cases:

i) When θ = 0°

τ =0 , ie, the torque is minimum when the plane of the loop is perpendicular to the magnetic field

ii) When = 90°

τ = NIAB ie, the torque is maximum when the plane of the loop is parallel to the magnetic field. Thus,

τ_{max} = NIAB